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Some Static Solutions of Electromagnetic and Scalar Field with Cylindrical Symmetry

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Abstract :

The present paper provides some static solutions of electromagnetic and scalar field in different cases for cylindrically symmetric space-time with two degrees of freedom. We have found value of non-zero component F_{14} of field tensor F_{ij}.

Key Words : Static, Scalar field, electromagnetic field, cylindrical symmetry, degree of freedom.

1. INTRODUCTION

Various researchers in general relativity have focused their mind to find static solution for cylindrically symmetric space time with electromagnetic field in recent years. Mishra and Radhakrishhna [9] studied some electromagnetic fields of cylindrical symmetry. Electromagnetic solutions of the field equations in general relativity has been obtained by Harrison [6]. Homogeneous solutions of Einstein-Maxwell's equations has been obtained by Rosen [16] and Ozsvath [11, 12]. Einstein – Maxwell's fields with non-zero charge current distribution has been studied by Goodinson and Newing [4]. Some properties of Einstein – Maxwell fields has been given by Nagraj [10]. Solutions of Einstein – Maxwell equations for cylindrically symmetric space time has already been extensively studied by Witten [25], Singh et al. [21], Roy and Tripathi [17]. Roy and Prakash [13] obtained some solutions of Einstein-Maxwell equations for a cylindrically symmetric space time with two degrees of freedom. Some other workers in this field are Yadav and Purushottam [27] and Zeldovich et. al. [28, 29].

Roy and Prakash [19] Considering the cylindrically symmetric of Marder [7], have constructed an isotropic magnetohydrodynamic coamological model in General Relativity. Latter on Singh and Yadav [22], have also constructed a non-static cylindrically symmetric commological model which is spatially homogeneous non-degenerate Petrov type I. They have assumed the energy momentum tensor to be that of a perfect fluid with an electromagnetic field. Sharma and Yadav [20] have considered the cylindrically symmetric metric with two degrees of freedom where the energy momentum tensor of a scalar field is associated with meson of rest mass μ. Singh et. al [23] have presented a procedure which enables one to construct solutions to the cylindrically symmetric Gravitational field coupled to electromagnetic and massless scalar fields. Some other workers in this line are Bati and Upadhayay [1], Bhar [2], Kumar [5], Mazumdar [8], Pradhan et. al. [14, 15], Rahman [18], Sinha and Sinha [26].

This paper presents some static solutions of electromagnetic and scalar field for cylindrically symmetric space time with two degrees of freedom in different cases.

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2. THE FIELD EQUATION

We take the metric of the space time in the form:

$$(2.1)\quad ds^2 = 2^{2\eta-2\delta}(dt^2 - dr^2)(x^2e^{2\delta} + r^2e^{2\varepsilon-2\delta})d\phi^2$$
$$-e^{2\delta}dz^2 - 2xe^{2\delta}d\phi\, dz$$

where η, x, δ and ε are functions of r only and r, ϕ, z, t correspond respectively x^1, x^2, x^3, x^4 co-ordinates. When $\varepsilon = 0$, this reduces to the stachal metric [17] with two degrees of freedom and when both $\varepsilon = 0$, $x = 0$ this goes to the well known Einstein-Rosen metric [1, 10] with one degree of freedom. The distribution consists of an electromagnetic field and a scalar zero rest mass meson field V. Thus the field equation are

$$(2.2)\quad R_{ij} = -K[E_{ij} + V_i, V_j],$$

$$(2.3)\quad E_{ij} = -g^{ab}F_{ai}F_{bj} + \frac{1}{4}g_{ij}F_{ab}F^{ab},$$

$$(2.4)\quad F_{[i,j,k]} = 0,$$

$$(2.5)\quad F^{ij}_{,j} = 0,$$

$$(2.6)\quad g^{ij}V_{;ij} = 0$$

For the metric (2.1) the components R_{12}, R_{13}, R_{24} and R_{34} vanish identically. This leads to

$$(2.7)\quad F_{13}F_{23}\frac{e^{2\delta-2\varepsilon}}{t^2} = F_{12}F_{23}\frac{xe^{2\delta-2\varepsilon}}{r^2} + F_{14}e^{2\delta-2\eta}$$

$$(2.8)\quad F_{12}F_{32}\left[e^{-2\delta} + x^2\frac{e^{2\delta-2\varepsilon}}{r^2}\right] = F_{13}F_{32}\frac{x}{r^2}e^{2\delta-2\varepsilon} + F_{14}F_{34}e^{2\delta-2\eta}$$

$$(2.9)\quad F_{31}F_{41}e^{2\delta-2\eta} = F_{32}F_{43}\frac{xe^{2\delta-2\varepsilon}}{r^2} = F_{32}F_{42}\left[e^{-2\delta} + \frac{x2e^{2\delta-2\varepsilon}}{r^2}\right]$$

$$(2.10)\quad F_{21}F_{41}e^{2\delta-2\eta} = F_{23}F_{42}\frac{x}{r^2}e^{2\delta-2\varepsilon} - F_{23}F_{43}\frac{1}{r^2}e^{2\delta-2\varepsilon}$$

In view of (2.7) – (2.10) two case say arise

(i) $F_{12} = F_{13} = F_{24} = F_{34} = 0$, F_{23}, $F_{14} \neq 0$

(ii) $F_{14} = F_{23} = 0$, F_{12}, F_{13}, F_{24}, $F_{34} \neq 0$

Case (i) is done directly in terms of F_{ij} components.

Case (ii) is done is terms of two potentials Q_2 and Q_3 which give rise to all the non-vanishing components of F_{ij}.

Here we will consider case (i) only.

3. SOME STATIC SOLUTIONS OF THE FIELD EQUATIONS

The field equations (2.2) – (2.6) for the metric (2.1) are

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(3.1a) $R_{11} = -K\left[-\frac{1}{2}\left\{e^{2\delta-2\eta}F_{14}^2 + \frac{e^{2\eta-2\delta-2\varepsilon}}{r^2}F_{23}^2\right\} + V_1^2\right]$

(3.1b) $R_{22} = -K\left[\frac{1}{2}P\{e^{4\delta-4\eta}(X^2e^{2\delta} + r^2e^{2\varepsilon-2\delta})F_{14}^2\right.$

$\left. +(X^2e^{2\delta-2\varepsilon} + r^2e^{-2\delta})\frac{F_{23}^2}{2r^2}\right]$

(3.1c) $R_{33} = -E\left[\frac{e^{6\delta-4\eta}}{2}F_{14}^2 + \frac{e^{2\delta-2\varepsilon}}{2r^2}F_{23}^2\right]$

(3.1d) $R_{44} = -E\left[\frac{1}{2}\left\{e^{2\delta-2\eta}F_{14}^2 + \frac{e^{2\eta-2\delta-2\varepsilon}}{r^2}r_{23}^2\right\}\right]$

(3.1e) $R_{23} = -K\left[\frac{X}{2}\left\{e^{6\delta-4\eta}F_{14}^2 + \frac{e^{2\delta-2\varepsilon}}{r^2}r_{23}^2\right\}\right]$

(3.2) $F_{23} = H$

(3.3) $F_{14} = K_1$

(3.4) $V_{11} + V_1\left(\frac{1}{r} + \varepsilon\right) = 0$

Equation (3.4) on integration yields

(3.5) $V = k_2\rho$

where we have defined the new variable ρ by means of the equation.

$$\frac{dr}{d\rho} = re^{\varepsilon}$$

The equations (3.1a) – (3.1e) are five equations in F_{14} and F_{23}. Eleminating F_{14} and F_{23} from these equations, we get the following four differential equations.

(3.6) $R_{11} + R_{44} + KV_1^2 = 0$

(3.7) $R_{11} + e^{2\eta-4\delta}R_{33} + KV_1^2 = 0$

(3.8) $R_{11} + \frac{e^{2\eta-2\varepsilon}}{r^2}R_{22} - \frac{xe^{2\eta-2\varepsilon}}{r^2}R_{23} + KV_1^2 = 0$

(3.9) $R_{23} - XR_{33} = 0$

Equations (2.6) – (2.9) give rise to the following differential equations.

(3.10) $\varepsilon_1^2 + 2\delta_1^2 + \varepsilon_{11} - 2\eta_1\varepsilon_1 + \frac{2\varepsilon_1}{r} - \frac{2\eta_1}{r} + X_1^2\frac{e^{4\delta-2\varepsilon}}{2r^2} + KV_1^2 = 0$

(3.11) $\varepsilon_1^2 + 2\delta_1^2 + \varepsilon_{11} - \eta_{11} - \varepsilon_1\eta_1 - \frac{\eta_1}{r} + \frac{2\varepsilon_1}{r} + KV_1^2 = 0$

(3.12) $$2\varepsilon_1^2+2\delta_1^2-\frac{2\delta_1}{r}-\frac{\eta_1}{r}+\frac{4\varepsilon_1}{r}+2\varepsilon_{11}+\eta_{11}-2\delta_{11}$$
$$-2\delta_1\varepsilon_1-\eta_1\varepsilon_1+\frac{4^{4\delta-2\varepsilon}}{r^2}\left[X_1^2+\frac{1}{2}\times X_{11}-\frac{1}{2}\times x_1\varepsilon_1+2\times x_1\delta_1-x\frac{x_1}{2r}\right]+KV_1^2=0$$

(3.13) $$2x_1\delta_1+\frac{1}{2}x_{11}-\frac{1}{2}\frac{x_1}{r}-\frac{1}{2}x_1\varepsilon_1=0$$

From equation (3.11), (3.12) and (3.13), we obtain

(3.14) $$\varepsilon_1^2-\frac{2\delta_1}{r}+\frac{2\varepsilon_1}{r}+\varepsilon_{11}-2\delta_{11}-2\delta_1\varepsilon_1+x_1^2\frac{e^{4\delta-2\varepsilon}}{r}=0$$

Equation (3.13) on integration gives

(3.15) $$\frac{x_1}{r}e^{4\delta-\varepsilon}=a$$

From equation (3.10) and (3.15) we obtain

(3.16) $$\eta_1=\frac{1}{2}\left(\varepsilon_1+\frac{1}{r}\right)+\frac{\varepsilon_1-\frac{1}{r^2}}{2\left(\varepsilon_1+\frac{1}{r}\right)}+\frac{\delta_1^2}{\left(\varepsilon_1+\frac{1}{r}\right)}$$
$$+\frac{a^2e^{-4\delta}}{4\left(\varepsilon_1+\frac{1}{r}\right)}+\frac{KV_1^2}{2\left(\varepsilon_1+\frac{1}{2}\right)}=0$$

From equation (3.14) and (3.15) we obtain

(3.17) $$\delta_{11}+\delta_{11}\left(\varepsilon_1+\frac{1}{r}\right)=\frac{1}{2}\left(\varepsilon_1^2+\frac{2\varepsilon_1}{r}+\varepsilon_{11}\right)+\frac{a^2e^{-4\delta}}{2}$$

From equation (3.10), (3.11) and (3.15) we get,

(3.18) $$\eta_{11}+\eta_1\left(\varepsilon_1+\frac{1}{r}\right)=\frac{a^2}{2}e^{-4\delta}$$

Equations (3.17) and (3.18) lead to

(3.19) $$\left(\delta_{11}-\eta_{11}\right)+\left(\delta_1-\eta_1\right)\left(\varepsilon_1+\frac{1}{r}\right)=\frac{1}{2}\left(\varepsilon_1+\frac{1}{r}\right)^2+\frac{1}{2}\left(\varepsilon_{11}-\frac{1}{r^2}\right)$$

which on integration yields

(3.20) $$\eta_1=\delta_1-\frac{\frac{1}{2}\left(re^{\varepsilon}\right)_1}{re^{\varepsilon}}-\frac{b}{re^{\varepsilon}}$$

where b is a constant of integration.

From equations (3.16) and (3.20) we obtain

(3.21) $$\left[\delta_1-\frac{1}{2}\left(\varepsilon_1+\frac{1}{r}\right)\right]^2+\frac{3}{4}\left(\varepsilon_1+\frac{1}{r}\right)^2+\frac{b}{re^{\varepsilon}}\left(\varepsilon_1+\frac{1}{r}\right)$$
$$+\frac{1}{2}\left(\varepsilon_{11}-\frac{1}{r^2}\right)+\frac{a^2}{4}e^{-4\delta}+\frac{KV_1^2}{2}=0$$

by the substitution

(3.22) $$\delta-\frac{1}{2}\varepsilon-\frac{1}{2}\log r=\alpha,$$

Equation (3.16) reduces to

(3.23) $$\alpha_{11}+\alpha_1\left(\varepsilon_1+\frac{1}{r}\right)-\frac{a^2e^{-4\alpha-2\varepsilon}}{2r^2}=0$$

which on integration gives

(3.24) $$\left[re^{\varepsilon}\alpha_1\right]^2=c_1^2-\frac{1}{4}a^2e^{-4\alpha}$$

c_1 being a constant of integration.

From equations (3.5), (3.21) and (3.24) we obtain

(3.25) $$\frac{1}{4}\left[\left(re^{\varepsilon}\right)_1\right]^2+b(re^{\varepsilon})_1+\frac{1}{2}(re^{\varepsilon})(re^{\varepsilon})_{11}+c_2^2=0$$

where $c_2^2=c_1^2+kk_1^2$

Let us now use the relation

$$re^{\varepsilon}=\frac{dr}{d\rho}$$

Equation (3.24) then leads to

(3.26) $$\frac{d\alpha}{\sqrt{e_1^2-\frac{1}{4}a^2e^{-4\alpha}}}=d\rho$$

which on integration gives

(3.27) $$e^{2\alpha}=\frac{a}{2c_1}\cos h[2c_1\rho-\beta]$$

Equations (3.22) and (3.27) lead to

(3.28) $$e^{4\delta}=\left[\frac{dr}{d\rho}\right]^2\frac{a^2}{4c_1}\cos h^2(2c_1\rho-\beta)$$

where s is a constant of integration and α is given by equation (3.27).

From equation (3.15) and (3.27) we obtain

(3.30) $$\frac{dX}{d\rho}=\frac{4c_1^2}{a}\sec h^2[2c_1\rho-\beta]$$

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which on integration gives

(3.31) $$X = \frac{2c_1}{a}\tanh[2c_1\rho - \beta] + \omega$$

ω being a constant of integration,

The substitutions

(3.32a) $re^{\varepsilon} = y$

and

(3.32b) $$\frac{dy}{d\rho} = p,$$

reduce equation (3.25) to

(3.33) $$\frac{dp}{dy} = \frac{p - 4bpy - 4c_2^2 y^2}{2py}$$

From equation (3.33) we have

(3.34) $$\frac{2v\,dv}{v^2 + 4bv + 4c_2^2} = -\frac{dy}{y}$$

where

(3.35) $p = vy$

Three cases arise

Case 1 : $b > c$

Equation (3.34) on integration gives

(3.36) $$y = N[v+L]^{\left[\frac{2L}{L-M}\right]}[v+M]^{\left[\frac{2M}{L-M}\right]}$$

where

$$L = 2\left[b + \sqrt{(b^2 - c_2^2)}\right]$$

and

$$M = 2\left[b - \sqrt{(b^2 - c_2^2)}\right]$$

N being a constant of integration.

Equations (3.32b), (3.34) and (3.35) lead to

(3.37) $$v = \frac{1}{2}\left[(L-M)\cot h\left\{\frac{(L-M)(n+\rho)}{8}\right\} - (L+M)\right]$$

n being a constant of integration.

From equation (3.32a), (3.36) and (3.37), we get

(3.38) $$r^{e^{\varepsilon}} = \frac{4N}{(L-M)^2}\exp\left[-\frac{(L+M)(n+\rho)}{4}\right]\sin h^2\left[\frac{(L-M)(n+\rho)}{8}\right]$$

Hence the metric of the space time is given by

(3.39) $$ds^2 = \exp\left[-\frac{1}{2}(L+M)\rho\right]\left[\frac{(L-M)^2}{4N}\right]\exp\left\{\frac{(L+M)(n+\rho)}{4}\right\}$$

$$\operatorname{cosec} h^2\left[\frac{(L-M)(n+\rho)}{8}\right]dt^2 - \frac{4N}{(L-M)^2}$$

$$\exp\left\{=\frac{(L+M)(n+\rho)}{4}\right\}\sin h^2\left[\frac{(L-M)(n+\rho)}{8}\right]d\rho^2$$

$$-\frac{2N}{(L-M)^2}\exp\left\{-\frac{(L+M)(n+\rho)}{4}\right\}\sin h^2\left[\frac{(L-M)(n+\rho)}{8}\right]$$

$$\frac{a}{c_1}\cos h(2c_1\rho-\beta)\left[\left\{\frac{2c_1}{a}\tan h(2c_1\rho-\beta)+\omega\right\}^2\right.$$

$$\left.+\frac{4c_1^2}{a^2}\sec h^2(2c_1\rho-\beta)\right]d\phi^2 - \frac{2N}{(L-M)^2}$$

$$\exp\left\{-\frac{(L+M)(n+\rho)}{4}\right\}\sin h^2\left[\frac{(L-M)(n+\rho)}{8}\right]$$

$$\frac{a}{c_1}\cos h(2c_1\rho-\beta)dz^2 - \frac{4N}{(L-M)}\exp\left\{-\frac{(L+M)(n+\rho)}{4}\right\}$$

$$\sin h^2\left[\frac{(L-M)(n+\rho)}{8}\right]\frac{a}{c_1}\cos h(2c_1\rho-\beta)$$

$$\left[\omega+\frac{2e_1}{a}\tanh(2c\rho-\beta)\right]d\phi\, dz.$$

By the following transformation

(4.3.40) $$\begin{cases}\rho=\rho\\ \sqrt{\dfrac{2Na}{c_1}}\dfrac{1}{L-M}\exp\left[-\dfrac{(L+M)n}{8}\right]z=Z\\ \sqrt{\dfrac{2Na}{c_1}}\dfrac{1}{L-M}\exp\left[-\dfrac{(L+M)n}{8}\right]\phi=\phi\\ \sqrt{\dfrac{s}{N}}\dfrac{L-M}{2}\exp\left[\dfrac{(L+M)n}{8}\right]t=T\end{cases}$$

The line element (3.39) takes the form

(3.41) $$ds^2 = \exp\left[-\frac{(L+M)}{4}\rho\right]\sin h^2\left\{\frac{(L-M)(n+\rho)}{8}\right\}$$

$$\left[\operatorname{cosec} h^4\left\{\frac{(L-M)(n+\rho)}{8}\right\}dT^2-\frac{4Ns}{(L-M)^2}\right.$$

$$\exp\left\{-\frac{(L+M)(n+2\rho)}{4}\right\}d\rho^2-\cos h\{2c_1\rho-\beta\}$$

$$\left\{\left(\left[\frac{2c_1}{a}\tan h(2c_1\rho-\beta)+\omega\right]^2+\frac{4c_1^2}{a^2}\sec h^2(2c_1\rho-\beta)d\phi^2\right.\right.$$

$$\left.\left.+dz^2+2\left[\frac{2c_1}{a}\tan h(2c_1\rho-\beta)+\omega\right]d\phi\ dZ\right\}\right]$$

The value of F_{14} is given by

$$(3.42)\quad F_{14}^2=\frac{(L-M)^2}{32\ K}dx\left[-\frac{(L+M)}{4}\right]$$

$$\operatorname{cosec} h^4\left[\frac{(L-M)(n+\rho)}{8}\right]\left[[1-\frac{32K}{(L-M)^2}H^2\right.$$

$$\left.\left.\exp\left\{-\frac{(L+M)}{4}\rho\right\}\operatorname{cosec} h^4\left\{\frac{(L-M)(n+\rho)}{8}\right\}\right]\right]$$

Case 2 : $b = c$

Equation (3.34) in integration givens

$$(3.43)\quad y=\frac{\ell}{(v+2c_2)}dxp\left[-\frac{4c_2}{v+2c_2}\right]$$

ℓ being a constant of integration,

From equations (3.32b), (3.34) and (3.35) we get

$$(3.44)\quad v=\frac{2}{m+\rho}=2c_2$$

where m is a constant of integration

Equation (3.32a), (3.43) and (3.44) lead to

$$(3.45)\quad re^{\varepsilon}=\frac{\ell}{4}(m+\rho)\exp\left[-2c_2(m+\rho)\right]$$

Consequently the line element takes the form

$$(3.46)\quad ds^2=\exp\left[\frac{4}{\ell}\frac{1}{(m+\rho)^2}e^{2c_2m}dt^2-\frac{1}{4}(m+\rho)^2\right.$$

$$\left.e^{-2c_2(m+2)}d\rho^2\right]-\frac{\ell a}{8c_1}dxp\left[-2c_2(m+\rho)\right]$$

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$$(m+\rho)^2 \cos h(2c_1\rho-\beta)\left[\left\{\frac{2c_1}{a}\tan h(2c_1\rho-\beta)+\omega\right\}^2\right.$$
$$\left.+\frac{4c_1}{a^2}\sec h^2(2c_1\rho-\beta)\right]d\phi^2$$
$$-\frac{\ell a}{8c_1}\exp\left[-2c_2(m+\rho)\right](m+\rho)^2\cos h(2c_1\rho-\beta)6z^2$$
$$-\frac{1a}{4c_1}\exp\left[-2c_2(m+\ell(m+\rho)^2\cos h(2c_1\rho-\beta)\right]$$
$$\left[\omega+\frac{2c_1}{a}\tan h(2c_1\rho-\beta)\right]d\phi\, dz.$$

The transformation

$$\rho=\rho$$

(3.47)
$$\begin{cases}\rho=\rho\\ \dfrac{1}{2\sqrt{2}}\sqrt{\dfrac{a1}{c_1}}\exp(-c_2m)z=Z\\ \dfrac{1}{2\sqrt{2}}\sqrt{\dfrac{a1}{c_1}}\exp(-c_2m)\phi=\phi\\ 2\sqrt{\dfrac{a}{1}}\exp(c_2m)t=T\end{cases}$$

Reduce the metric (3.46) to the form

(3.48)
$$ds^2=\left[\frac{1}{(m+\rho)^2}dT^2-\frac{1a}{4}(m+\rho)^2\right]\exp\left[-2c_2(m+2\rho)d\rho^2\right]$$
$$-\exp(-2c_2\rho)(m+\rho)^2\cos h(2c_1\rho-\beta)\left[\left\{\left[\frac{2c_1}{a}\tan h(2c_1\rho-\beta+\omega\right]^2\right.\right.$$
$$\left.+\frac{4c_1^2}{a^2}\sec h^2(2c_1\rho-\beta)\right\}d\phi^2+dZ^2$$
$$\left.+2\left\{\frac{2c_1}{a}\tan h(2c_1\rho-\beta\right\}d\phi\, dZ\right]$$

The value of F_{14} for the space time (3.48) is given by

(3.49)
$$F_{14}^2=\frac{2}{k}\frac{1}{(m+\rho)^4}-\frac{H^2}{(m+\rho)^8}$$

Case 3 : $b < c$

Equation (3.34) on integration gives

(3.50) $$y = \left[\frac{Q}{(v+2b)^2 + 4(c_2^2 - b^2)}\right] \left[\exp \frac{2b}{\sqrt{c^2 - b^2}} \tan^{-1}\left(\frac{v+2b}{2\sqrt{c_2^2 - b^2}}\right)\right]$$

D being a constant of integration.

From equations (3.32b), (3.34) and (3.35) we have

(4.3.51) $$v = 2\sqrt{c_2^2 - b^2}\left[\tan\left\{\sqrt{c_2^2 - b^2}(\lambda - \rho)\right\}\right] - 2b$$

where λ is a constant of the integration.

From equations (3.32a), (3.50) and (3.51) we get

(3.52) $$re^{\varepsilon} = \frac{Q}{4(c_2^2 - b^2)} dxp\left[-2b(\rho - \lambda)\right] \cos^2\left[\sqrt{c_2^2 - b^2}(\rho - \lambda)\right]$$

Consequently the metric of the space time can be cast into the form.

(3.53) $$\begin{aligned} ds^2 = s\Bigg[& \frac{4(c_2^2 - b^2)}{Q} \exp(-2b\lambda) \sec^2\left\{\sqrt{\left(c_2^2 - b^2\right)}(\lambda - \ell)\right\} dt^2 \\ & - \frac{Q}{4(c_2^2 - b^2)} \exp[2b(\lambda - 2\ell)] \cos^2\left\{\sqrt{c_2^2 - b^2}(\lambda - \ell\right\} d\rho^2 \Bigg] \\ & - \frac{aQ}{8c_1(c_2^2 - b^2)} \cosh(2c_1\rho - \beta) \exp\left[2b(\lambda - \rho) \cos^2\left\{\sqrt{c_2^2 - b^2}(\lambda - \ell)\right\}\right. \\ & \left[\frac{2c_1}{a} \tan h(2c_1\rho - \beta) + \omega^2 + \frac{4c_1^2}{a^2} \sec h^2(2c_1\rho - \beta)\right] d\phi^2 \\ & - \frac{aQ}{8c_1(c_2^2 - b^2)} \cos(2c_1\rho - \beta) \exp[2b(\lambda - \rho)] \cos^2\{\sqrt{c_2^2 - b^2}(\lambda - \ell)\} dz^2 \\ & - \frac{aQ}{4c_1(c_2^2 - b^2)} \cos h(2c_1\rho - \beta) \exp[2b(\lambda - \ell)] \cos^2\left\{\sqrt{c_2^2 - b^2}(\lambda - \rho)\right\} \\ & \left[\frac{2c_1}{a} \tan h(2c_1\rho - \beta + \omega\right] d\phi\, dz \end{aligned}$$

$$[\because \cos(-\theta) = \cos\theta]$$

The transformation

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$$\rho = \rho$$

(3.54) $$\frac{\sqrt{a}/\sqrt{Q}}{2\sqrt{2c_1}\ \sqrt{c_2^2 - b^2}} \exp(b\lambda) z = Z$$

$$\frac{\sqrt{a}\sqrt{Q}}{2\sqrt{2c_1}\sqrt{c_2^2 - b^2}} \exp(b\lambda)\phi = \phi$$

$$\frac{2\sqrt{s}\ \sqrt{c_2^2 - b^2}}{\sqrt{Q}} \exp(-b\lambda) t = T$$

reduce the line element (3.53) to the form

(3.55) $$ds^2 = \sec^2\left\{\sqrt{c_2^2 - b^2}\ (\rho - \lambda)\right\}[dT^2 - \frac{Qs}{4(c_2^2 - b^2)}$$

$$\exp\left[-2b(2\rho - \lambda)\right]\cos^4\left\{\sqrt{c_2^2 - b^2}(\rho - \lambda)\right\}d\rho^2]$$

$$-\exp\left(-2b\rho\right)\cosh(2c_1\rho - \beta)\cos^2\left\{\sqrt{c_2^2 - b^2}(\rho - \lambda\right\}$$

$$\left[\{[\frac{2c_1}{a}\tan h(2c_1\rho - \beta) + \omega]^2 + \frac{4c_1^2}{a^2}\sec h^2(2c_1\rho - \beta)\}d\phi^2 + dZ^2\right.$$

$$\left. + 2\left\{\frac{2c_1}{a}\tan h(2c_1\rho - \beta) + \omega\right\}d\phi\ dZ\right]$$

For which

(3.56) $$F_{14}^2 = \frac{2}{k}(c_2^2 - b^2)\sec^4\left[\sqrt{c_2^2 - b^2}(\rho - \lambda)\right]$$

$$\left[1 - \frac{K\ H^2}{2(c_2^2 - b^2)}\sec h^4\left\{\sqrt{c_2^2 - b^2}(\rho - \lambda)\right\}\right]$$

4. DISCUSSION

In all the three cases if we put $k_2 = 0$ (or $c_2 = c_1 = c$) i.e. If zero-rest mass scalar meson field V is absent the solution of Roy and Prakash [13] is obtained.

APPENDIX :

The components of Ricci tensor R_{ij} are given by

$$R_{11} - \varepsilon_1^2 + 2\delta_2^2 - \frac{1}{r}\left(\delta_1 + \eta_1 - 2\varepsilon_1\right) + \varepsilon_{11} + \eta_{11} - \delta_{11}$$

$$= \delta_1\varepsilon_1 - \eta_1\varepsilon_1 + \frac{1}{2}X_2^2\frac{e^{4\delta - 2\varepsilon}}{r^2}$$

$$R_{33} = e^{4\delta - 2\eta}\left[\delta_{11} + \frac{\delta_1}{r} + \delta_1\varepsilon_1\right] - \frac{1}{2r^2}e^{8\delta - 2\eta - 2\varepsilon}\chi_1^2$$

$$R_{23} = e^{4\delta-2\eta}\left[2\chi_1\delta_1 + \frac{1}{2}\chi_{11} + \chi_{11} - \frac{1}{2}\frac{\chi_1}{r} + \frac{\chi\delta_1}{r}\right.$$

$$\left.-\frac{1}{2}\chi_1\varepsilon_1 + \chi\delta_1\varepsilon_1\right] - \frac{1}{2r^2}\chi x_1^2\, e^{8\delta-2\eta-2\varepsilon}$$

$$R_{22} = 2\chi\ R_{23} - \left(\chi^2 + r^2 e^{2\varepsilon-4\delta}\right) R_{33}$$

$$R_{44} = \delta_{11} - \eta_{11} + \frac{1}{r}\left(\delta_1 - \eta_1\right) + \delta_1\varepsilon_1 - \eta_1\varepsilon_1$$

The components of E_{ij} are given by

$$-E_{11} = E_{44} = \frac{1}{2}\left[e^{2\delta-2\eta}F_{14}^2 + \frac{e^{2\eta-2\delta-2\varepsilon}}{r^2}F_{23}^2\right]$$

$$E_{22} = \left(\chi^2 e^{2\delta} + r^2 e^{2\varepsilon-2\delta}\right) e^{2\delta-2\eta} E_{44}$$

$$E_{33} = e^{4\delta-2\eta}\, E_{44}$$

$$E_{23} = \chi\, E_{33}$$

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